REALISTIC APPROACH AND DESIGN OF SLENDER REINFORCED CONCRETE COLUMNS - A CASE STUDY

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ABSTRACT

Slender Reinforced concrete columns are subjected to significant lateral deformation, fails mainly by buckling, due to development of secondary moment. Reinforced Concrete columns with larger height with respect to lateral dimension are subjected to significant lateral deformation on application of external load (either axial or eccentrical) and subsequently develop secondary moment. This moment induces additional deflection and hence there is an increase in secondary moment. As a result the load-moment curve becomes non-linear. It is obvious that due to this secondary moment the load carrying capacity of the column is reduced. Prediction of ultimate load for reinforced concrete slender columns involves study of buckling through material non-linearity and cracking behaviour of cross section, since the failure occurs in inelastic range. The present design methods of reinforced concrete slender columns suggested by American, British and Indian code provisions are either empirical or involve cumbersome procedure. To circumvent the above, a simple, realistic and rational theory is proposed, incorporating the behaviour of reinforced concrete slender columns rectangular in cross section, bent in single curvature. The theory also incorporates the material non-linearity and effect of cracking at the time of failure. An experimental investigation is undertaken to validate the theory developed. In addition, design charts are prepared based on the theory and a realistic design procedure is proposed for practical applications.

Key Words: Slender, Reinforced Concrete Column, effective flexural stiffness

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1. INTRODUCTION

A slender Reinforced concrete column is obvious that due to this secondary moment the load carrying capacity of the column is reduced (Mac Gregor et al. 1970). In existing practice, the analysis is carried out by various methods like Moment Magnifier Method, Reduction Factor Method and Additional Moment Method (Mac Gregor et al. 1970). The loads thus obtained from first order analysis are modified to account for slenderness effect. Many authors (Guralinic, S.A. and Swartz,E. 1969, Guralinic, S.A. and Suresh Desai, 1970, Mac Gregor et al. 1970, Poun-Hewi Chaung et al. 1998) have proposed different procedures to assess the strength of slender columns and to design slender columns. But generally they have used the modulus for concrete as suggested by ACI Specifications and the moment of inertia as either transformed section or modified section (Poon-Hwei Chaung et al. 1998), based on the empirical formula incorporating the stability and cracking behaviour of the column. However, a rational approach for the moment of inertia calculation will be, to incorporate the cracking and variation of cracking along the height of the column.

In general the current methods either lack rationalism or simplicity in accounting the effect of lateral deflection, non-linear material characteristics of concrete and cracking of the section in the analysis and design of slender RCC columns. Instead they are either largely empirical or tend to be too complex for every day design office use (Vijaya Rangan 1990). Hence, the aim of this paper is to present a rational, realistic and a simple method to design slender RCC columns which will also find easy application among design engineers.

Among many researchers who had suggested design procedures for the design of slender columns, pioneering contribution had been made by Mac Gregor (1970,1993,1996), B.VijayaRangan (1990), Guralinic, S.A (1969 & 1970), Swartz,E (1969), Suresh Desai (1970), Poon-Hwei Chaung and Sia-Keong Kong (1998).

The theory proposed by Guralinic, S.A. and Swartz, E (1969) is based on the use of transformed section to calculate moment of inertia (I_{tr}) and the modulus of concrete is calculated based on ACI (1963) provisions, which are empirical in nature. Also the procedure is restricted with only two cases i.e., Slender column bent in single curvature due to equal end moments or eccentricities applied simultaneously, since the test data available are not sufficient for other complex cases. Further Guralinic,S.A and Suresh Desai (1970), suggested another procedure based on the moment magnification concept, incorporating second order theory. In which, the central deflection is calculated using Fourier series (Timoshenko, 1961). This procedure correctly predicts the collapse load and modes of failure, but a generalized procedure to analyse the slender column for complicated cases such as column with unequal end moments, column with double curvature, side sway etc., is not developed.

Vijaya Rangan (1990) suggested a method based on simplified stability analysis incorporating all the general features including creep. But in this procedure also the modulus of concrete and moment of inertia calculations are more empirical.

In the numerical method evaluated by Poon-Hwei Chaung et al (1998) to analyse slender reinforced concrete columns, based on transformation concept, a constant modulus for concrete is obtained from the different secant moduli of reinforcing steel and concrete (Carrasquillo et al.1981) Here, Transformed section is used to calculate the moment of inertia (I_{tr}) which is again empirical.

The effective flexural stiffness (EI) of a slender reinforced column is strongly affected by cracking and its variation over the height at the time of buckling and inelastic behaviour of concrete and reinforcing steel. Flexural stiffness (EI) is, therefore a function of many variables and does not lend itself to very simple analytical equations as suggested by American code (2002) British (1997) and Indian (2000) standards. Hence it is required to develop a theory

based on a realistic modulus value of the material at the instant of buckling. Out of various moduli suggested by researchers such as Young's modulus, Tangent modulus, Double Modulus, Secant modulus etc., (Alexander Chajes, 1974), a suitable flexural rigidity is to be arrived at, such that it will truly incorporate the non-linear material characteristics as well as cracking behaviour of the concrete section. An experimental investigation is carried out to validate the proposed theory and a design procedure is suggested to design reinforced concrete slender column incorporating all the above parameters.

2. THEORY

A theory has been proposed and published (Parameswaran.P *et al.* 2004) by the author to analyse a pin-ended slender reinforced concrete column subjected to axial load with initial imperfections/eccentricity and deforming in single curvature, incorporating the material and geometrical non-linearity of the column. This theory provides a simple, realistic and rational approach to find the strength of the slender column based on the stability criteria. It is assumed that the critical strain occurs at the point of bifurcation and a suitable flexural rigidity (EI) values is evaluated based on the non-linear stress-strain characteristics of the material and the effective cross section at the time of failure.

To account for the material non-linearity in concrete, out of many stress-strain relations available, the following (Levi 1961) expression is used for its general acceptance.

$$f_c = \frac{0.85 \times f_{cc}}{\varepsilon_o^2} \left(2 \times \varepsilon \times \varepsilon_o - \varepsilon^2 \right)$$

(1)

where,

f_{cc} = Ultimate 28 days cylinder compressive strength of concrete

ε_o = Concrete strain corresponding to stress f_{cc}

ε = Maximum strain in concrete.

To calculate the buckling load, use of tangent modulus in evaluating the flexural rigidity (EI) value is generally accepted by many researchers (Chajes 1974), even though it leads to conservative values. But, this is not valid in the descending part of the stress-strain curve, since it yields negative values in this zone. Hence, to achieve still more realistic modulus value even in drooping portion also, secant modulus is adopted in this approach and is given by,

$$E_S = \frac{f_c}{\varepsilon} = \frac{0.85 \times f_{cc}}{\varepsilon_o^2} \left(2\varepsilon_o - \varepsilon \right)$$

(2)

At the time of buckling, tensile cracks are developed over the cross section and gross moment of inertia is not valid and hence it is to be modified accordingly. By knowing the position of neutral axis (X_u), the net cross section under compression can be calculated. From Fig.10, it is clear that the locus of the position of neutral axis 'X_u' for various sections along the length of the column will be parabolic or straight line in nature depending on the values of e/D, d´/D, strength of concrete and percentage of steel. Hence the effective depth of the section can be arrived as,

$$D_{eff} = D_{net} + \frac{1}{3} \left[D_{gr} - D_{net} \right], \text{ for parabolic variation}$$

(3)

$$D_{eff} = D_{net} + \frac{1}{2} \left[D_{gr} - D_{net} \right], \text{ for straight line variation}$$

(4)

But in general for practical problem, to be on conservative side, effective depth can be assumed as given in Eq. (3). Hence the effective moment of inertia is given by,

$$I_{eff} = \frac{bD_{eff}^{3}}{12} \qquad (5)$$

Critical strain in slender column will occur when the column deforms from straight configuration to that of adjacent bent configuration i.e., point of bifurcation. Hence, the critical strain can be evaluated by equating the strength of the section to corresponding Euler's critical load at the time of bifurcation and is given by (Parameswaran.P *et al.* 2004),

Critical strain, $\varepsilon_{cr} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ \qquad (6)

where,

$$a = 1 - p$$

$$b = -\left(0.004(1-p) + 0.9412\frac{p}{f_{cc}} + \frac{9.8696}{\lambda^2}\right)$$

$$c = \frac{0.0395}{\lambda^2}$$

Note: If the length of the column is short such that ε_{cr}, as determined from Eq. (6) is greater than or equal to ε_o (i.e., 0.002), then the load carrying capacity of the column, P_{cr} is controlled by material failure and must be evaluated as per the strength considerations of a short column.

In practice, no column exists without imperfections/eccentricity. The effects of an imperfect column can also be studied (Chajes 1974) by considering a straight but eccentrically loaded member, assuming that the member is initially straight, the material obeys Hooks law and the deformation remains small. At the time of application of load, deformation will increase and by equating the internal resisting moment at midheight of the column to the corresponding applied moment, the deformation at the midheight of the column can be evaluated. The deflection due to eccentricity is given by (Chajes 1974),

$$\delta = e\left[Sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right) - 1\right] \qquad (7)$$

Using the Eqs.(1)-(6), critical load for any slender column subjected to axial load with or without eccentricity can be calculated as,

$$P_{cr} = \frac{\pi^2 E_S . I_{eff}}{L_{eff}^{2}} \qquad (8)$$

Note: If a column has double curvature initially, its strength will be more than that of same column bent in single curvature, but at the time of failure, the column bends only in single curvature and hence the strength of the column gets reduced suddenly. However, since the theory is based on single curvature bending, the results from the theory are always conservative.

3. EXPERIMENTAL PROGRAMME

An investigation was undertaken to study the behaviour and strength of concrete column subjected to axial load with or without eccentricity. The results thus obtained were compared with the theory (Parameswaran.P *et al.* 2004) developed based on stability criteria and conclusions are drawn. Totally ten number of columns (both axially and eccentrically loaded)

with different slenderness values (l/d) varying from 8 to 50 were cast and tested. The slenderness ratios were selected such that the columns fall in the categories of short, slender and very slender columns.

3.1. Materials

A nominal mix to yield M 20 grade concrete, as specified by IS 456-2000 was used. OPC 43 grade cement, natural river sand and crushed graded aggregate of maximum size 10mm were used. HYSD steel with 0.2% proof strength of 415 MPa was used for main reinforcement and lateral ties.

3.2. Casting of Specimen

The moulds were prepared with plywood, in order to achieve accuracy in dimensions of the specimens. The ends of the columns were widened and provided with suitable bearing plate and reinforcement (Fig.1) to ensure uniform distribution of loading and to prevent local failure at the support during testing. The column specimens were cast horizontally in the moulds and thoroughly vibrated by means of a needle vibrator. The columns were de-moulded on the next day and cured with wet gunny bags. The specimen details are as indicated in Table1. Control specimens such as cubes (150mm×150mm×150mm) and cylinders (150mm diameter × 300 mm height) were cast along with every specimen and cured at the same condition as that of specimen.

3.3. Test Setup

UTM of capacity 1000 kN modified with suitable attachments particularly for testing columns (Fig.2) was used to test the column specimens. Ball and socket arrangement was provided at the top and a hinge was provided at bottom end. The verticality of the columns were checked with plumb bob. Columns were tested with varying load eccentricities, with a value of 0.05D for CC groups and 0.25D for EC groups. Ball and socket assembly ensures that the load eccentricity is maintained at all stages of loading. LVDTs were placed at uniform vertical intervals as shown in Fig.2, to measure the longitudinal deflections of the column. Electric Resistant Strain gauges were affixed on both faces of column at the midheight to measure the strain variations.

3.4. Test Procedure

The loads were applied axially with minimum eccentricity (0.05D) and required eccentricity (0.25D). An initial set load of 5 kN was applied and released to zero in each test and then initial readings were observed. The loads were applied gradually with uniform increment till the column failed. For each increment of load longitudinal deflections at various heights and strain at midheight of both faces were measured.

4. RESULT AND DISCUSSION

4.1. Failure Loads and Failure Modes

A summary of test data is given in Table 2 which gives ultimate load, mid height deflection at ultimate load and maximum strain for the test specimen. The average compressive strength (f_{ck}) of the specimen is 40.1 MPa. As expected, the increase in slenderness ratio decreased the ultimate load carrying capacity of the column. The failure load was also dependent on the load eccentricity. An increase in the load eccentricity resulted in a decrease in failure load and increase in mid height deflection at failure (CC group and EC group). In general, the column failed at midheight or close to mid height.

Generally, when a column is subjected to axial load, it fails either due to crushing or buckling depending on the slenderness effect, material properties, magnitude and eccentricity of the applied load and its end conditions. If a column is perfectly straight and short and is subjected to axial load and if the magnitude of the eccentricity is very small such that it does not develop any appreciable bending moment, it fails by crushing. In such failures, concrete fails by crushing and shearing outwards along the inclined planes in addition to the vertical bursting cracks due to the tensile stress developed in outward direction (Nilson 2001).

Whereas a slender column, whether it is subjected to axial force with or without imperfection/eccentricity, exhibits large lateral deformation, leading to a buckling failure (Fig.3). In such cases, failure will be initiated by spalling of the cover concrete in the compression zone (Fig.4) and flexural cracks will develop and gradually extend inside the section along the tension face and finally an unbound deformation will take place. Sometimes, depending on the level of imperfection and eccentricities present at the ends, columns with very high slenderness ratio may deflect in double curvature and when the load is increased gradually the deflection on one part of the column gets reversed and becomes single curvature deflection before failure, which is called as a phenomenon of reversal of deflection (Timoshenko and Gere 1961). This occurs because of the non-linear relation between the deflection and the compressive force.

In the present study some of the above modes of failures were observed. Short and intermediate columns with minimum eccentricity or imperfections (CC1, CC2 and EC1) did not show much lateral deflection (less than 0.05D) and exhibited typical failure. Columns with larger eccentricity (CC5 and EC5) had shown sheet spalling of the cover concrete in compression zone. This kind of sheet spalling behaviour of failure in very slender columns with larger load eccentricities was already observed by Lloyd and Rangan (1996) in their experiments. Column EC5 had shown a typical reversal of deflection (Timoshenko and Gere 1961) phenomenon as discussed above. All the other columns (slender in nature) failed in flexure with spalling of cover concrete in the compression zone and tensile cracks along the tension face. The mid height deflection observed in these columns varied from 0.09D to a maximum of 0.24D depending upon the slenderness ratio, initial imperfection and load eccentricity. When the critical load is reached the deflection became unbound and which led to buckling failure of the column.

4.2. Load- Deflection Curves

Fig.5 and Fig.6 illustrates typical load-deflection curves at midheight for the tested columns. The curves show the ductile behaviour of the columns. For the eccentrically loaded columns the deflections are found to be predominant which is an important criteria in stability problems, hence the load eccentricity being a significant parameter. The following general features are observed;

- Short or intermediate Column with small load eccentricity show minimum deformation and the failure is initiated by the sudden spalling of cover concrete in compression zone.

- Slender columns with large load eccentricity exhibited greater deformations at failure load. Very slender column with load eccentricity developed tensile cracks with unbound deformations prior to failure.

4.3. Moment-Curvature Curves

The curvature at midheight of the column was calculated based on the strain measurements taken and moment curvature curves were plotted, and found that, these curves have the same trend that of the load-deformation curves and it is inferred that when the load is increased, the

column loses its flexural rigidity. The ultimate strain values at failure compare well with the critical strain calculated from the theory as observed in Table 3.

4.4. Comparison of Predicted and Experimental Critical Loads

South well plot is generally an accepted procedure to evaluate critical load for columns that fail within elastic limit. But in the present study short and intermediate columns were also tested which failed beyond elastic limit and hence this method cannot be used here. Kwon and Hancock's (1992) procedures to evaluate critical load are briefed here.

Critical load can be calculated by plotting the load against the square of deflection and subsequently fitting a line through the test data in post buckling region. The intersection of the fitted line and the initial tangent can be taken as the critical load. From Fig.5 it is found that fitting a straight line, which, falls on at least three readings, to draw a proper tangent line in the post-buckling region, is very difficult and hence this procedure also cannot be used in the present study.

Another procedure suggested by Kwon and Hancock (1992) to evaluate critical load is the intersection of the initial tangent and the fitted line in the post-buckling region of the plot between log of load versus the lateral deflection (Fig.7). Critical loads thus calculated are tabulated in Table 3 and show a good agreement with the theoretical critical load calculated.

Columns CC1 and EC1 fall in the category of short column and the strength of the columns are based on the strength criteria. Similarly column EC5 exhibits reversal of deflection phenomena (Double curvature). Hence, the critical load calculated based on the theory is more conservative.

5. DESIGN APPLICATIONS

A rational and simple design procedure is developed to design a slender column subjected to axial load and eccentricity based on the above theory incorporating the realistic flexural rigidity values and the effect of slenderness ratio. The following design charts were developed to make the design easier.

- l/r Vs. Critical strain   (Fig.8)

- l/r Vs. $P_{cr}/f_{cc}bD$ (Fig.9)

- e/D Vs. X_u/D (Fig.10)

- e/D Vs. P/P_E (Fig.11)

6. CONCLUSIONS

A method is presented for predicting the ultimate load of a column subjected to axial load with uniaxial eccentricity, pinned at both ends and free of side sway. The pin ended concentrically loaded column may be treated as a special case of the general procedure. This method is substantiated by means of a set of experimental investigations conducted for different slenderness ratios. The following conclusions are made from the experimental investigations.

- Slender columns with imperfections/eccentricity initially bend in single or double curvature, but in case of double curvature, when the load is increased gradually, the phenomenon of reversal of deflection takes place and failed in single curvature.

- Short column with imperfections generally, subjected to compressive stress throughout its cross section, failed due to crushing, whereas slender column failed within elastic limit due to buckling.

- Very slender columns with large load eccentricity developed tensile cracks with unbound deformations prior to failure.

- Maximum strain at the time of failure is well predicted by the theory in the case of more slender columns.

- Experimental critical loads has good correlation with the theoretical critical loads, hence the present theory can be validated.

- Design charts are developed for easy design applications.

- Because of the simplicity in the theory developed using design charts, for predicting ultimate loads, this procedure may be adopted to the field directly.

Table 1 Specimen Details

Column No.	Dimensions (mm)			l/b	Main Reinforcement	Stirrups
	b	D	l			
CC1	150	150	1200	8	4 nos. of 10 mm dia.	8 mm dia. @ 150 mm c/c
EC1						
CC2	75	100	1500	20	4 nos. of 8 mm dia.	6 mm dia. @ 75 mm c/c
EC2						
CC3	60	100	1800	30	4 nos. of 8 mm dia.	6 mm dia. @ 60 mm c/c
EC3						
CC4	50	100	2000	40	4 nos. of 8 mm dia.	6 mm dia. @ 50 mm c/c
EC4						
CC5	40	100	2000	50	4 nos. of 8 mm dia.	6 mm dia. @ 40 mm c/c
EC5						

Table 2 Observations

Column No.	Eccentricity (mm)	Ultimate Load (kN)	Ultimate deflection at midheight (mm)	Ultimate strain in compression face ($\times 10^{-6}$)
CC1	7.50	595	2.28	1.545
CC2	3.75	325	2.34	1.274
CC3	3.00	130	9.45	0.690
CC4	2.50	87.5	10.79	0.350
CC5	2.00	37.5	24.13	0.220
EC1	37.50	320	4.98	1.835
EC2	18.75	216	13.92	1.542
EC3	15.00	109	10.79	0.883
EC4	12.50	75	5.07	0.435
EC5	10.00	112.3	10.96	0.192

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**Table 3** Correlation of experiment and predicted strength

Column No.	Ultimate Load (kN)	Critical load, P_e (kN) from Log P Vs. δ	Theoretical critical load, P_{cr} (kN)	Critical Strain ($\times10^{-3}$) Experiment, ε_e	Theoretical, ε_{cr}	P_e/P_{cr}	$\varepsilon_e/\varepsilon_{cr}$
CC1	595	595.0*	621.0	1.845	2.000	*	0.923
CC2	325	251.0	241.3	1.374	1.585	1.042	0.867
CC3	130	120.22	116.47	0.690	0.722	1.032	0.956
CC4	87.5	63.10	59.97	0.350	0.400	1.052	0.875
CC5	37.5	20.89	23.71	0.220	0.242	0.881	0.909
EC1	320	320.0*	342.0	1.835	2.000	*	0.918
EC2	216	138.1	130.0	1.542	1.585	1.062	0.973
EC3	109	57.54	51.72	0.883	0.722	1.113	1.222
EC4	75	28.82	24.05	0.435	0.400	1.199	1.088
EC5	112.3	87.00**	-	**	0.242	**	**
				Mean		1.054	0.970
				Coefficient of Variation		0.091	0.119
				Standard Deviation		0.096	0.116

* Short column

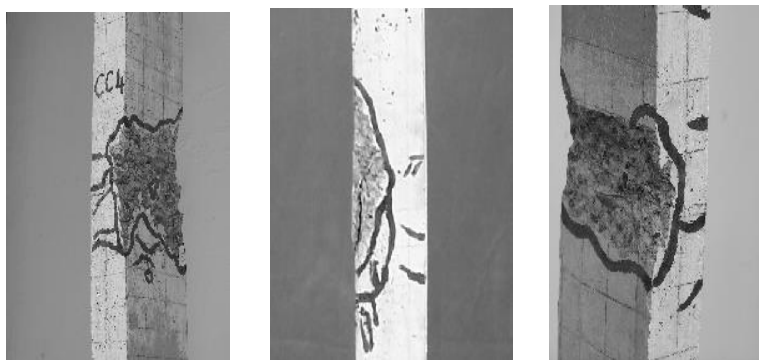*** Double curvature failure*
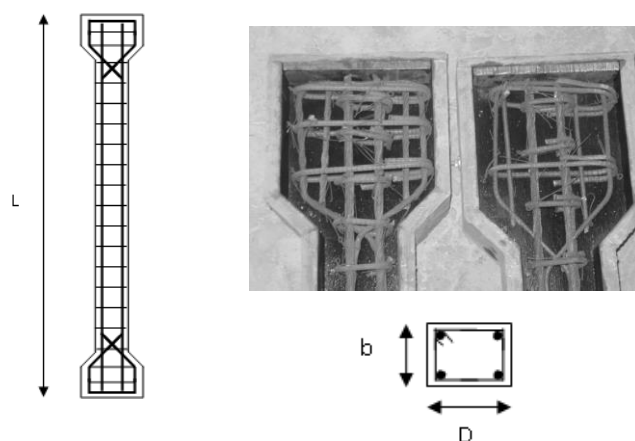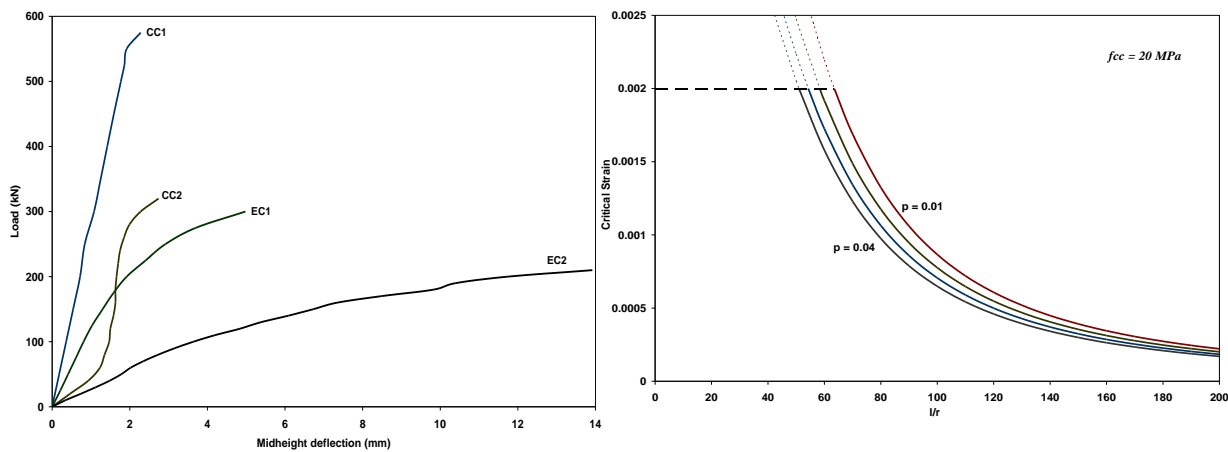

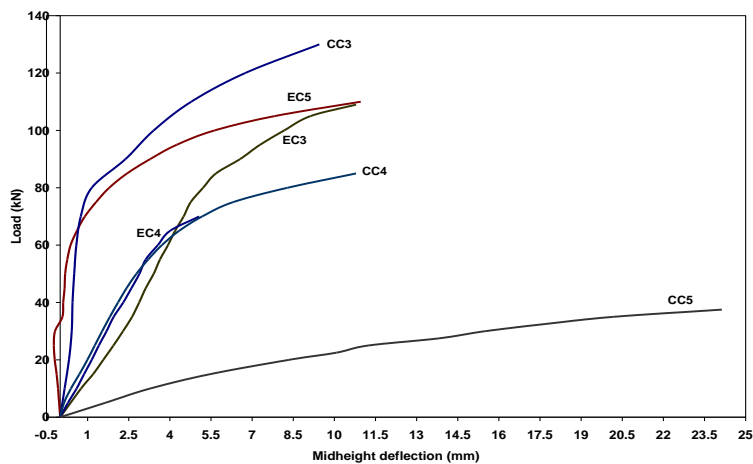**Figure 1** Geometry and reinforcement details
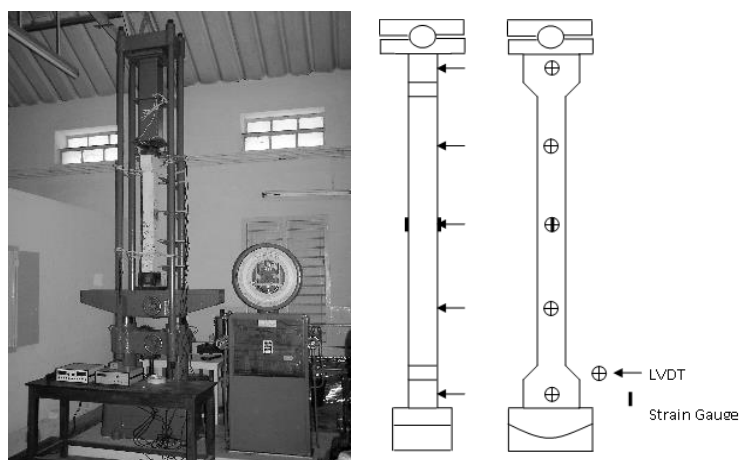
Figure 2 Test Setup
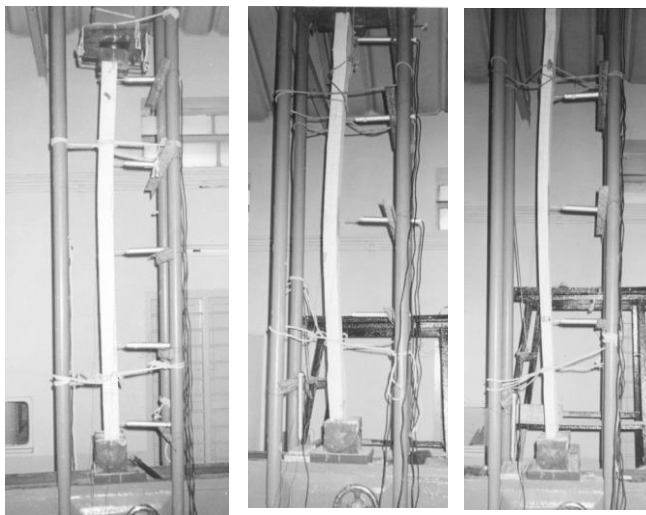
Figure 3 Buckling of columns
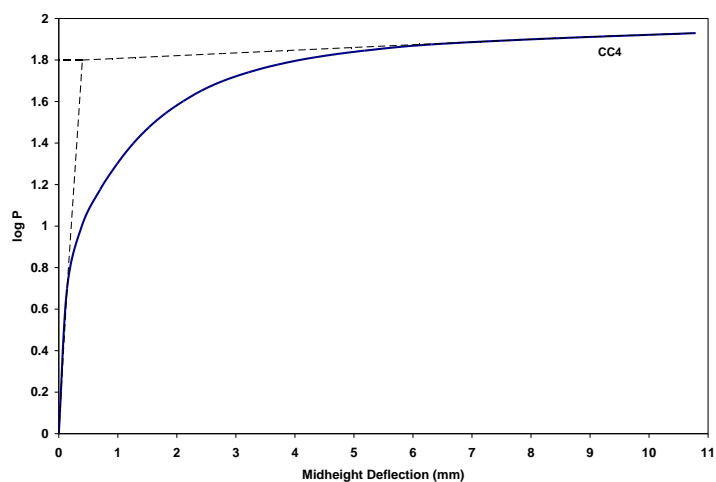

Figure 4 Modes of failure
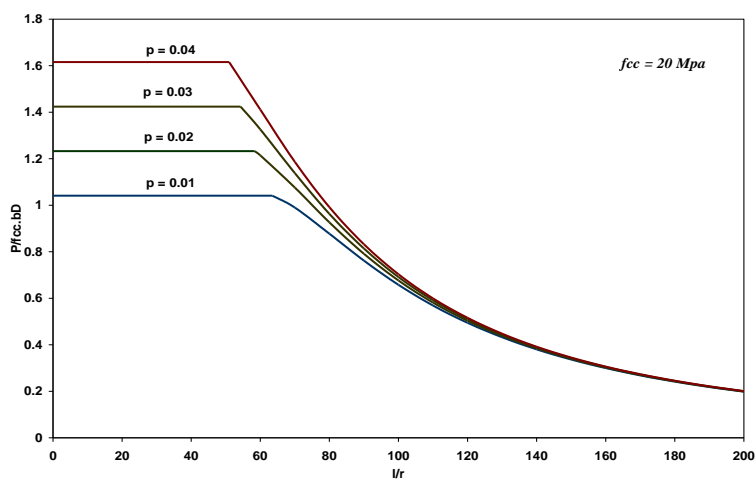

Figure 5 Load Vs. Midheight Deflection (Columns with l/r ≤ 20)

Figure 6 Load Vs. Midheight Deflection (Columns with l/r > 20)

Figure 7 Experimental Critical load - Log P Vs. δ (CC4)

Figure 8 Slenderness ratio Vs. $P_{cr}/f_{cc}bD$
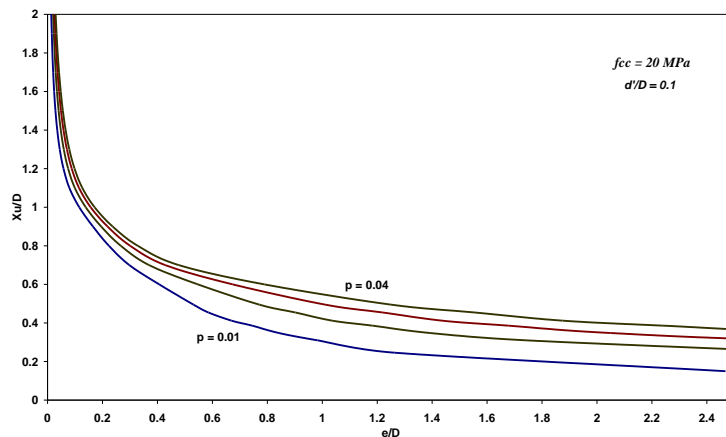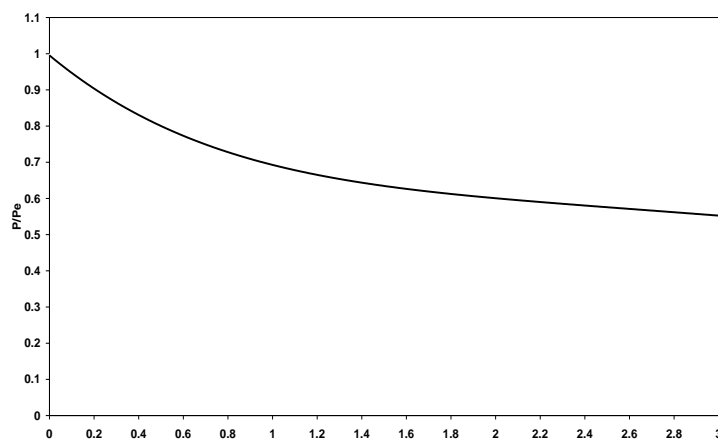
Figure 9 e/D Vs. X_u/D

Figure 10 e/D Vs. P/P_E

Nomenclature:

D_{eff} =   Effective depth

D_{gr} =   Depth of cracked section at midheight

D_{net} =   Depth of gross section at midheight

E_s =   Secant modulus of concrete

L_{eff} =   Effective length of the column

P_E =   Euler's buckling load

P_{cr} =   Critical buckling load of the column

P_e =   Experimental critical buckling load of column

X_u =   Depth Neutral axis

e =   Eccentricity

f_{cc} =   Ultimate 28 days cylinder compressive strength of concrete

p =   Percentage ratio

ε =   Maximum strain in concrete.

ε_{cr} =   Critical Strain

$\varepsilon_o =$ Concrete strain corresponding to stress f_{cc}

$\delta \ =$ Deflection at the mid section

$\lambda \ =$ Slenderness ratio $\left(L_{eff} \middle/ r \right)$

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